MATHEMATICS SYLLABUS

Pre-University

Higher 2

Syllabus 9758

Implementation starting with
2024 Pre-University One Cohort

Ministry of Education
SINGAPORE

Contents

SECTION 1: INTRODUCTION

Importance of Learning Mathematics
Mathematics at the A-Level
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1. Introduction

Importance of Learning Mathematics

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

Mathematics at the A-Level

There are four syllabuses to cater to the different needs, interests, and abilities of students:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

H2 Mathematics is designed to prepare students for a range of university courses, including mathematics, sciences and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

Assumed knowledge: G3 Additional Mathematics

Students without G3 Additional Mathematics (e.g. those who offered G2 Additional Mathematics or just G3 Mathematics) may offer H2 Mathematics but will be required to bridge the knowledge gap during the course of study.

Learning mathematics at the A-Level provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills. It also exposes students to a way of thinking that complements other ways of thinking developed through the other disciplines.

2024 A-Level Mathematics Curriculum

The 2024 A-Level Mathematics Curriculum is an update of the 2016 A-Level Mathematics Curriculum and incorporates the key shifts in the 2020 Secondary Mathematics Curriculum. All the syllabuses will continue to emphasise the development of mathematical processes and highlight applications of mathematics. In addition, the 2024 A-level Mathematics Curriculum will emphasise the following:

a) *Strengthening mathematical practices*: These are practices that enable students to seek problems and learn mathematics on their own, construct knowledge and communicate their ideas mathematically. More opportunities should be created for students to be engaged in such practices, which also support the development of 21st century competencies.

b) *Using computers as mathematical tools*: This goes beyond the use of computers for teaching and learning, but for doing mathematical work. It creates opportunities for students to develop computational thinking that is also encouraged at the secondary level. Learning objectives that explicitly mention the use of computers and software/apps as mathematical tools are included, where appropriate, but will <u>not</u> be examinable.

c) *Teaching towards big ideas*: This will strengthen students' appreciation and deepen their understanding of mathematics, and will encourage students to see beyond the topics, and also their connections. It is a continuation from their learning experiences at the secondary level, where teaching towards big ideas is emphasised. The themes and big ideas that are featured in the 2024 A-Level Mathematics Curriculum are described in Section 2.

SECTION 2: MATHEMATICS CURRICULUM

Nature of Mathematics
Themes and Big Ideas
Mathematics Curriculum Framework
Mathematics and 21st Century Competencies

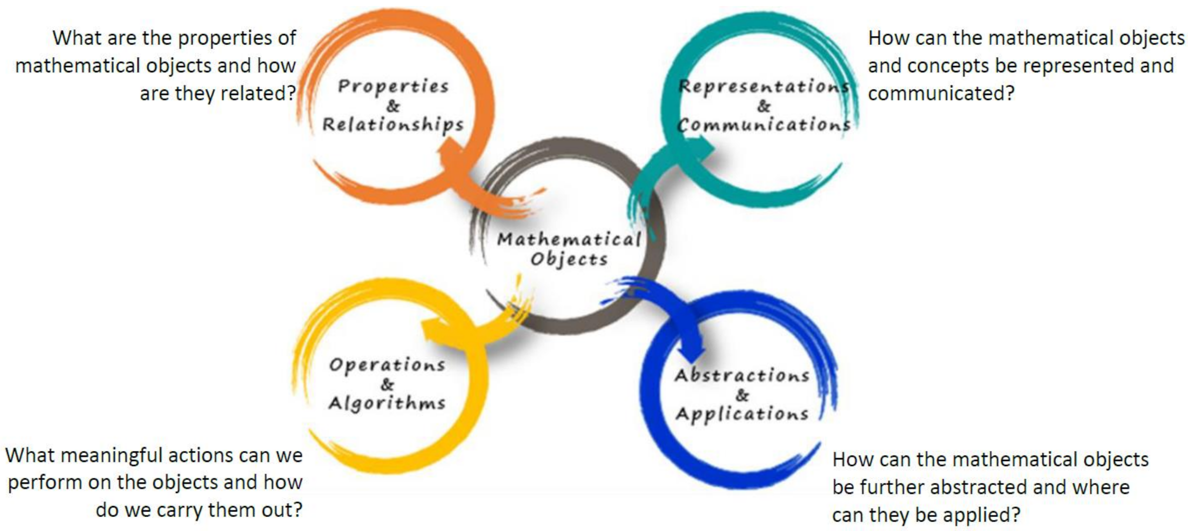
2. Mathematics Curriculum

Nature of Mathematics

Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

Themes and Big Ideas

From the above description of the nature of mathematics, four recurring themes in the study of mathematics are derived: *Properties and Relationships*, *Operations and Algorithms*, *Representations and Communications*, and *Abstractions and Applications*.



1. *Properties and Relationships*: What are the properties of mathematical objects and how are they related?

   Properties of mathematical objects (e.g. numbers, lines, function, etc.) are either inherent in their definitions or derived through logical argument and rigorous proof. *Relationships* exist between mathematical objects. They include the equivalence of two expressions or statements, the connections between two functions, relationship between vector equations of lines and planes, and relationship between independent and dependent variables. Understanding properties and relationships enable us to gain deeper insights into the mathematical objects and use them to model and solve real-world problems.

2. *Operations and Algorithms*: What meaningful actions can we perform on the mathematical objects and how do we carry them out?

   Operations are meaningful actions performed on mathematical objects. They include algebraic manipulations, geometric transformations, operations on functions, and many more. *Algorithms* are generalised sequences of well-defined smaller steps to perform a mathematical operation or to solve a problem. An example is the root-finding algorithms to approximate the roots of an equation (e.g. using Newton-Raphson method) or solutions of first order differential equations (e.g. using Euler method). Understanding the meaning of these operations and algorithms and how to carry them out enable us to solve problems mathematically.

3. *Representations and Communications*: How can the mathematical objects and concepts be represented and communicated within and beyond the discipline?

   Representations are integral to the language of mathematics. They include symbols, notations, and diagrams such as graphs, geometrical figures, Venn diagrams and tree diagrams that are used to express mathematical concepts, properties and operations in a way that is precise and universally understood. *Communication* of mathematics is necessary for the understanding and dissemination of knowledge within the community of practitioners as well as general public. It includes clear presentation of proof in technical writing as well as choosing appropriate representations (e.g. stating null and alternative hypotheses, using a scatter diagram to represent relationship between two variables) to communicate mathematical ideas that can be understood by the masses.

4. *Abstractions and Applications*: How can the mathematical objects be further abstracted and where can they be applied?

   Abstraction is at the core of mathematical thinking. It involves the process of generalisation, extension and synthesis. Through algebra, we generalise arithmetic. Through complex numbers, we extend the number system. Through coordinate geometry, we synthesise the concepts across the algebra and geometry strands. The processes of abstraction make visible the structure and rich connections within mathematics and makes mathematics a powerful tool. *Application* of mathematics is made possible by abstractions. From simple counting to complex modelling, the abstract mathematical objects, properties, operations, relationships and representations can be used to model and study real-world phenomena.

Big ideas express ideas that are *central* to mathematics. They appear in different topics and strands. There is a *continuation* of the ideas across levels. They bring *coherence* and show *connections* across different topics, strands and levels. The big ideas in mathematics could be about one or more themes, that is, it could be about *properties and relationships* of mathematical objects and concepts and the *operations and algorithms* involving these objects and concepts, or it could be about *abstraction and applications* alone. Understanding the big ideas brings one closer to appreciating the nature of mathematics.

Eight clusters of big ideas are listed in 2024 A-Level Mathematics Curriculum. Each cluster of big ideas is represented by a label for ease of reference: **Functions**, **Diagrams**, **Models**, **Equivalence**, **Transformation**, **Limits**, **Vectors**, and **Extensions**. They relate to the four themes that cut across and connect concepts from the different content strands. Some big ideas extend across and connect more concepts than others, and some also extend from the big ideas in the 2020 Secondary Mathematics Curriculum[1]. The list of big ideas is not meant to be authoritative or comprehensive.

A brief description of the big ideas in the 2024 A-Level Mathematics Curriculum is given below.

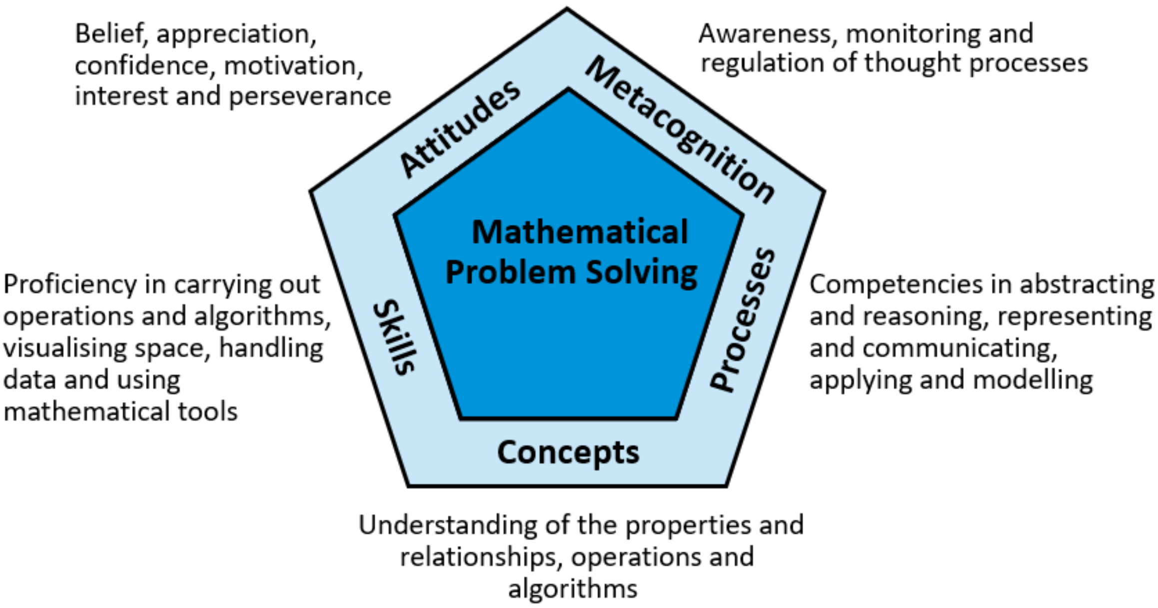
FUNCTIONS (Main Theme: Properties and Relationships) Functions express the relationship between two sets of mathematical objects by a rule that maps the elements of one set to those in the other set. The rule of a function may be expressed verbally, algebraically, numerically (as a table) or graphically (as a graph). Functions may have inverse and can be combined. Many operations and algorithms in mathematics can be thought of as a function, with appropriate input, rule and output. The input and output need not be limited to real numbers. This conceptualisation of function is useful when thinking about how to implement an operation or algorithm as codes in computer.	**DIAGRAMS** (Main Theme: Representations and Communications) Real-world or mathematical objects can be represented succinctly and visually using mathematical diagrams. Diagrams serve to communicate properties of the objects, show the relationship between objects and facilitate problem solving. Understanding what different diagrams represent, their features and conventions, and how they are constructed helps us understand and communicate mathematical ideas and results.
MODELS (Main Theme: Abstractions and Applications) Real-world objects and phenomena can be represented mathematically as models. Models are often approximations or simplifications and have limitations and assumptions. They may be deterministic or probabilistic and could be derived from theory or data. They enable us to describe patterns, analyse situations, predict outcomes and make decisions in those realistic contexts.	**EQUIVALENCE** (Main Theme: Properties and Relationships) Equivalence is a relationship that expresses the 'equality' of two mathematical objects (e.g. expressions, equations, statements) that may be represented in different forms. The transformation or conversion from one form to another equivalent form is the basis of many manipulations for analysing and comparing them as well as algorithms for finding solutions.

[1] *Functions*, *Diagrams*, *Models*, and *Equivalence* are also big ideas in the 2020 Secondary Mathematics Curriculum.

TRANSFORMATION (Main Theme: Operations and Algorithms) Transformation refers to changes made to a mathematical object using a clearly defined rule. When an object (e.g. graph, equation, or random variable) is transformed, its properties may or may not change (i.e. invariant). Understanding the nature and effects of these transformations enables us to develop insights into the relationships between the transformed object and the original object and to use these relationships to develop methods to solve problems.	**LIMITS** (Main Theme: Properties and Relationships) Limits describe the behaviour of a mathematical object (e.g. a model) that varies with a parameter as the parameter approaches a certain value or infinity. Limits may or may not exist. Both cases provide insights to prove mathematical results, justify the appropriateness of algorithms to obtain approximate value of an exact solution or explain local, long-term or large-scale behaviour.
VECTORS (Theme: Representations and Communications) Vectors are ordered array of numbers. They are higher dimensional generalisation of numbers, which are used to measure or quantify a property of a mathematical object. In its concrete form, vectors are used to describe points, lines and planes in geometry. In its abstract form, a dataset or polynomial can be represented as a vector. Vectors are ways of representing a mathematical object that requires more than one quantity or dimension to specify.	**EXTENSIONS** (Theme: Abstractions and Applications) Extensions of a mathematical object, concept or result widen its applicability. Extensions are common in the study of mathematics and is a means by which further abstraction, generalisation and applications can be achieved in mathematics.

Mathematics Curriculum Framework

The central focus of the mathematics curriculum is the development of mathematical problem solving competency. This also includes the curiosity to pose problems and the ability to make conjecture. Supporting this focus are five inter-related components – *concepts*, *skills*, *processes*, *metacognition* and *attitudes*.



- *Mathematical Problem Solving*

Problems may come from everyday contexts or future work situations, in other areas of study, or within mathematics itself. They include straightforward and routine tasks that require selection and application of the appropriate concepts and skills, as well as complex and non-routine tasks that requires deeper insights, logical reasoning and creative thinking. General problem solving strategies e.g. Polya's 4 steps to problem solving and the use of heuristics, are important in helping one tackle non-routine tasks systematically and effectively.

- *Concepts*

The understanding of mathematical concepts, their properties and relationships and the related operations and algorithms, are essential for solving problems. In the A-Level mathematics curriculum, concepts in functions and graphs, sequences and series, vectors, calculus, probability and statistics, and so on, are explored. These content strands are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

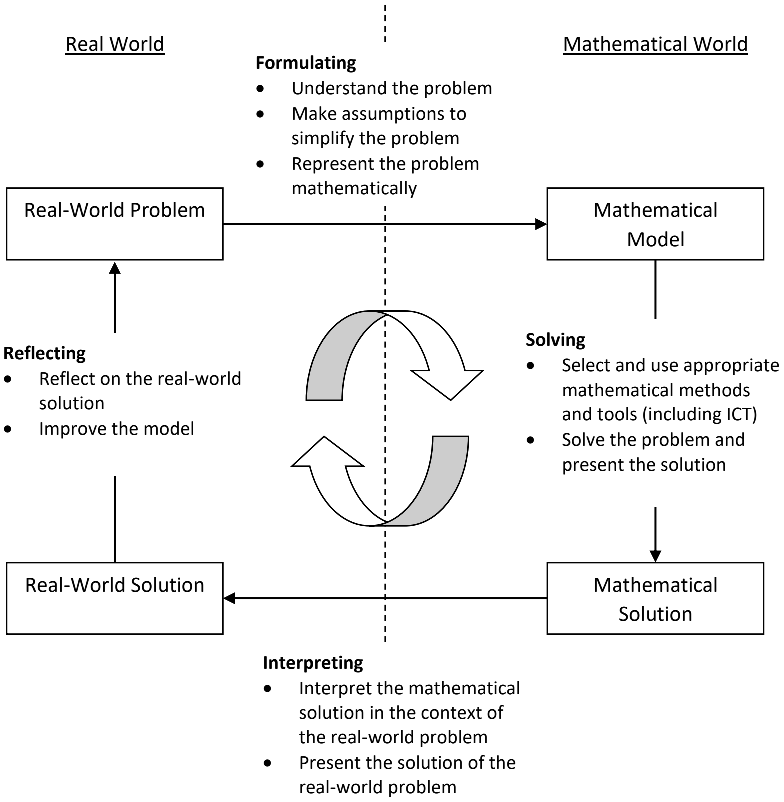
- *Skills*

Being proficient in carrying out the mathematical operations and algorithms and in visualising space, handling data and using mathematical tools (including spreadsheets and graphing applications) are essential for solving problems. In the A-Level mathematics curriculum, operations and algorithms such as *calculation, estimation, manipulation*, and *simplification* are required in most problems.

- *Processes*

Mathematical processes refer to the practices of mathematicians and users of mathematics that are important for one to solve problems and build new knowledge. These include abstracting, reasoning, representing and communicating, applying and modelling. Abstraction is what makes mathematics powerful and applicable. Justifying a result, deriving new results and generalising patterns involve reasoning. Expressing one's ideas, solutions and arguments to different audiences involves representing and communicating, and using the notations (symbols and conventions of writing) that are part of the mathematics language. Applying mathematics to real-world problems often involves modelling, where reasonable assumptions and simplifications are made so that problems can be formulated mathematically, and where mathematical solutions are interpreted and evaluated in the context of the real-world problem. [The mathematical modelling process is shown in the diagram below.]

Mathematical Modelling Process



- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring and regulation of one's own thinking and learning. It also includes awareness of one's affective responses towards a problem. When one is engaged in solving a non-routine or open-ended problem, metacognition is required.

- *Attitudes*

Having positive attitudes towards mathematics contributes to one's disposition and inclination towards using mathematics to solve problems. Attitudes include one's belief and appreciation of the value of mathematics, one's confidence and motivation in using mathematics, and one's interests and perseverance to solve problems using mathematics.

Mathematics and 21st Century Competencies

The learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century, in particular, *Critical, Adaptive and Inventive Thinking*. For example, when students pose questions, justify claims, write and critique mathematical explanations and arguments, they are engaged in not only mathematical reasoning and communication, but also critical thinking. When students devise different strategies to solve an open-ended problem or formulate different mathematical models to represent a real-world problem, they are engaged in inventive thinking. When students vary their approaches to solve different but related problems, they are engaged in adaptive thinking.

As an overarching approach, the A-Level mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21st century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, adaptively and inventively, reason logically and communicate effectively, work individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population, health and sustainability issues can help students understand the challenges faced by Singapore and those around the world.

SECTION 3:

H2 MATHEMATICS SYLLABUS

Preamble
Syllabus Aims
Content Strands
Applications and Contexts
Content

3. H2 Mathematics (From 2024)

Preamble

Mathematics is a basic and important discipline that contributes to the developments and understandings of the sciences and other disciplines. It is used by scientists, engineers, business analysts and psychologists, etc. to model, understand and solve problems in their respective fields. A good foundation in mathematics and the ability to reason mathematically are therefore essential for students to be successful in their pursuit of various disciplines.

H2 Mathematics is designed to prepare students for a range of university courses, such as mathematics, sciences, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

Syllabus Aims

The aims of H2 Mathematics are to enable students to:

a) acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines;
b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;
c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and
d) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

Content Strands

There are 6 content strands in H2 Mathematics, namely, *Functions and Graphs*, *Sequences and Series*, *Vectors*, *Introduction to Complex Numbers*, *Calculus,* and *Probability and Statistics*.

a) <u>Functions and Graphs</u> provides a more abstract treatment of functions and their properties and studies the characteristics of a wider class of graphs including graphs defined parametrically, as well as transformation of graphs, techniques for solving equations, inequalities and system of equations.

b) <u>Sequences and Series</u> provides a useful tool for describing changes in discrete models and includes special series and sequences such as arithmetic and geometric progressions, and the concepts of convergence and infinity.

c) Vectors provides a useful tool for physical sciences as well as a means to describe and work with objects such as points, lines and planes in two- and three-dimensional spaces.

d) Introduction to Complex Numbers provides a short introduction to complex numbers as an extension of the number system and includes complex roots of polynomial equations, the four operations and the representation of complex numbers in the Argand diagram.

e) Calculus provides useful tools for analysing and modelling change and behaviour and includes extension of differentiation and integration from Additional Mathematics, with additional techniques and applications such as power series and differential equations.

f) Probability and Statistics provides the foundation for modelling chance phenomena and making inferences with data and includes an introduction to counting techniques, computation of probability, general and specific discrete distribution models, normal distributions, sampling and hypothesis testing as well as correlation and regression.

Applications and Contexts

As H2 Mathematics is designed for students who intend to pursue a university course that is mathematics-related such as science and engineering, students should therefore be exposed to the applications of mathematics in science and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study. The list below illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied, and is by no means exhaustive.

Applications and contexts	Some possible topics involved
Kinematics and dynamics (e.g. free fall, projectile motion, collisions)	Functions; Calculus; Vectors
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Population growth, radioactive decay, heating and cooling problems, viral spread	Differential equations
Financial Maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Games of chance	Probability
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies), medical diagnosis, forecasting	Probability (conditional probability and independence); Sampling distributions; Hypothesis testing; Correlation and regression

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

Content

(Note: Learning Objectives that are italicised are non-examinable.)

	Topics/ Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	**Functions and Graphs**	
1.1	Functions	Include: • concepts of function, domain and range • inverse functions and composite functions • conditions for the existence of inverse functions and composite functions • domain restriction to obtain an inverse function • relationship between graphs of a one-to-one function and its inverse Exclude the use of the relation $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$, and restriction of domain to obtain a composite function.
1.2	Graphs and transformations	Include: • use of a graphing calculator *or a graphing software* to graph a given function • important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following: $y^2 = ax$; $x^2 = by$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ $y = \frac{ax + b}{cx + d}$ $y = \frac{ax^2 + bx + c}{dx + e}$ • equations of asymptotes, axes of symmetry, and restrictions on the possible values of x and/or y • effect of transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ and $y = \mathrm{f}(ax)$ and combinations of these transformations • relating the graphs of $y = \lvert\mathrm{f}(x)\rvert$, $y = \mathrm{f}(\lvert x\rvert)$, and $y = \frac{1}{\mathrm{f}(x)}$ to the graph of $y = \mathrm{f}(x)$ • simple parametric equations and their graphs

	Topics/ Sub-topics	Content
1.3	Equations and inequalities	Include: • formulating an equation, a system of linear equations, or inequalities from a problem situation • solving an equation exactly or approximately using a graphing calculator *or a graphing software* • solving a system of linear equations using a graphing calculator *or a graphing software* • solving inequalities of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)} > 0$ where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are linear expressions or quadratic expressions • concept of $\|x\|$, and use of relations $\|x-a\| < b \Leftrightarrow a-b < x < a+b$ and $\|x-a\| > b \Leftrightarrow x < a-b$ or $x > a+b$ • solving inequalities by graphical methods
2	**Sequences and Series**	
2.1	Sequences and series	Include: • concepts of sequence and series for finite and infinite cases • sequence as function $y = \mathrm{f}(n)$ where n is a positive integer • relationship between u_n (the nth term) and S_n (the sum to n terms) • sequence given by a formula for the nth term • sequence generated by the relation $u_{n+1} = \mathrm{f}(u_n)$, including the use of a graphing calculator *or a computer* to generate the sequence • sum and difference of two series • convergence of a series and the sum to infinity • formula for the nth term and the sum of a finite arithmetic series • formula for the nth term and the sum of a finite geometric series • condition for convergence of an infinite geometric series • formula for the sum to infinity of a convergent geometric series
3	**Vectors**	
3.1	Basic properties of vectors in two- and three-dimensions	Include: • addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations • position vectors, displacement vectors and direction vectors • magnitude of a vector • unit vectors • distance between two points • collinearity • use of the ratio theorem in geometrical applications

	Topics/ Sub-topics	Content
3.2	Scalar and vector products in vectors	Include: • concepts of scalar product and vector product of vectors and their properties • angle between two vectors • geometrical meanings of $\lvert\mathbf{a}\cdot\hat{\mathbf{n}}\rvert$ and $\lvert\mathbf{a}\times\hat{\mathbf{n}}\rvert$, where $\hat{\mathbf{n}}$ is a unit vector Exclude triple products $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$.
3.3	Three-dimensional vector geometry	Include: • vector and cartesian equations of lines and planes • foot of the perpendicular and distance from a point to a line or to a plane • angle between two lines, between a line and a plane, or between two planes • relationships between (i) two lines (coplanar or skew) (ii) a line and a plane (iii) two planes Exclude: • shortest distance between two skew lines • common perpendicular to two skew lines
4	**Introduction to Complex Numbers**	
4.1	Complex numbers expressed in Cartesian form and Argand diagrams	Include: • extension of the number system from real numbers to complex numbers • complex roots of quadratic equations • modulus, argument and conjugate of a complex number • four operations of complex numbers • equality of complex numbers • conjugate roots of a polynomial equation with real coefficients • representation of complex numbers in the Argand diagram • geometrical effects of conjugation, negation, addition, subtraction, and multiplication by i Exclude complex numbers expressed in polar (or modulus-argument) form and exponential form.
5	**Calculus**	
5.1	Differentiation	Include: • graphical interpretation of (i) $\mathrm{f}'(x)>0, \mathrm{f}'(x)=0$ and $\mathrm{f}'(x)<0$ (ii) $\mathrm{f}''(x)>0$ and $\mathrm{f}''(x)<0$ • relating the graph of $y=\mathrm{f}'(x)$ to the graph of $y=\mathrm{f}(x)$ • differentiation of simple functions defined implicitly or parametrically

	Topics/ Sub-topics	Content
		• determining the nature of the stationary points (local maximum and minimum points and points of inflexion) analytically, in simple cases, using the first derivative test or the second derivative test • locating maximum and minimum points using a graphing calculator *or a graphing software* • finding the approximate value of a derivative at a given point using a graphing calculator *or a graphing software* • problems involving tangents and normals to curves, including cases where the curve is defined implicitly or parametrically • local maxima and minima problems • connected rates of change problems Exclude non-stationary points of inflexion and finding second derivative of functions defined parametrically.
5.2	Maclaurin series	Include: • standard series expansion of $(1+x)^n$ for any rational n, e^x, $\sin x$, $\cos x$ and $\ln(1+x)$ • derivation of the first few terms of the Maclaurin series by - repeated differentiation, e.g. $\sec x$ - repeated implicit differentiation, e.g. $y^3+y^2+y=x^2-2x$ - using standard series, e.g. $\mathrm{e}^x\cos 2x$, $\ln\left(\frac{1+x}{1-x}\right)$ • range of values of x for which a standard series converges • concept of Maclaurin's series as an approximation of a function • small angle approximations: $\sin x \approx x$, $\cos x \approx 1-\frac{1}{2}x^2$, $\tan x \approx x$ Exclude problems involving derivation of the general term of a series.
5.3	Integration techniques	Include: • integration of $\mathrm{f}'(x)[\mathrm{f}(x)]^n$ (including $n=-1$), $\mathrm{f}'(x)\mathrm{e}^{\mathrm{f}(x)}$ $\sin^2 x$, $\cos^2 x$, $\tan^2 x$ $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{a^2-x^2}$ and $\frac{1}{x^2-a^2}$ • integration by a given substitution • integration by parts Exclude reduction formulae.

	Topics/ Sub-topics	Content
5.4	Definite integrals	Include: • concept of definite integral as a limit of sum • definite integral as the area under a curve • evaluation of definite integrals • area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • area below the x-axis • volume of revolution about the x- or y-axis • finding the approximate value of a definite integral using a graphing calculator *or a graphing software* Exclude area and volume of revolution about the x-axis or y-axis where curve is defined parametrically.
5.5	Differential equations	Include: • solving for the general solutions and particular solutions of differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)\mathrm{g}(y)$, including reducing a given differential equation to this form by means of a given substitution • formulating a differential equation from a problem situation • interpreting a differential equation and its solution in terms of a problem situation
SECTION B: PROBABILITY AND STATISTICS		
6	**Probability and Statistics**	
6.1	Probability	Include: • addition and multiplication principles for counting • concepts of permutation (${}^{n}\mathrm{P}_{r}$) and combination (${}^{n}\mathrm{C}_{r}$) • arrangements of objects in a line or in a circle, including cases involving repetition and restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A\|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

	Topics/ Sub-topics	Content
6.2	Discrete random variables	Include: • concept of discrete random variables, probability distributions, expectations and variances • concept of binomial distribution $\text{B}(n, p)$ as an example of a discrete probability distribution and use of $\text{B}(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of binomial distribution (without proof) Exclude finding cumulative distribution function of a discrete random variable.
6.3	Normal distribution	Include: • *concept of continuous random variables* • concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $\text{N}(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $\text{P}(X < x_1)$ or a related probability, given the values of $x_1,\ \mu,\ \sigma$ • symmetry of the normal curve and its properties • finding a relationship between $x_1,\ \mu,\ \sigma$ given the value of $\text{P}(X < x_1)$ or a related probability • solving problems involving the use of $\text{E}(aX + b)$ and $\text{Var}(aX + b)$ • solving problems involving the use of $\text{E}(aX + bY)$ and $\text{Var}(aX + bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.
6.4	Sampling	Include: • concepts of population and simple random sample • concept of the sample mean $\overline{X}$ as a random variable with $\text{E}(\overline{X}) = \mu$ and $\text{Var}(\overline{X}) = \frac{\sigma^2}{n}$ • distribution of sample mean from a normal population • use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. $n \geq 30$) • use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form $\sum x$ and $\sum x^2$, or $\sum(x - a)$ and $\sum(x - a)^2$

<table>
<tr><th></th><th>Topics/ Sub-topics</th><th>Content</th></tr>
<tr><td>6.5</td><td>Hypothesis testing</td><td>Include:
• concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance, and *p*-value
• formulation of hypotheses and testing for a population mean based on:
- a sample from a normal population of known variance
- a large sample from any population
• 1-tail and 2-tail tests
• interpretation of the results of a hypothesis test in the context of the problem

Exclude the use of the term 'Type I error', concept of Type II error and testing the difference between two population means.</td></tr>
<tr><td>6.6</td><td>Correlation and Linear regression</td><td>Include:
• use of scatter diagram to judge if there is a plausible linear relationship between the two variables
• correlation coefficient as a measure of the fit of a linear model to the scatter diagram
• interpreting the product moment correlation coefficient (in particular, values close to -1, 0 and 1)
• concepts of linear regression and method of least squares to find the equation of the regression line
• concepts of interpolation and extrapolation
• use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model
• use of a square, reciprocal or logarithmic transformation to achieve linearity

Exclude:
• problems involving derivation of formulae
• relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients
• hypothesis tests</td></tr>
</table>

Assumed Knowledge from O-Level/G3 Additional Mathematics

<table>
<tr><td colspan="2">Content from O-Level Additional Mathematics</td></tr>
<tr><td colspan="2">ALGEBRA</td></tr>
<tr><td>A1,
A2</td><td>Quadratic functions; Equations and inequalities
• Finding the maximum or minimum value of a quadratic function using the method of completing the square
• Conditions for a quadratic equation to have:
(i) two real roots
(ii) two equal roots
(iii) no real roots
• Conditions for $ax^2 + bx + c$ to be always positive (or always negative)
• Solving simultaneous equations in two variables by substitution, with one of the equations being a linear equation</td></tr>
<tr><td>A3</td><td>Surds
• Four operations on surds, including rationalising the denominator
• Solving equations involving surds</td></tr>
<tr><td>A4</td><td>Polynomials and partial fractions
• Multiplication and division of polynomials
• Use of remainder and factor theorems
• Partial fractions with cases where the denominator is not more complicated than:
- $(ax + b)(cx + d)$
- $(ax + b)(cx + d)^2$
- $(ax + b)(x^2 + c^2)$</td></tr>
<tr><td>A6</td><td>Exponential and logarithmic functions
• Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs, including:
- laws of logarithms
- equivalence of $y = a^x$ and $x = \log_a y$
- change of base of logarithms
• Simplifying expressions and solving simple equations involving exponential and logarithmic functions</td></tr>
<tr><td colspan="2">GEOMETRY AND TRIGONOMETRY</td></tr>
<tr><td>G1</td><td>Trigonometric functions, identities and equations
• Six trigonometric functions, and principal values of the inverses of sine, cosine and tangent
• Trigonometric equations and identities (see List of Formulae)
• Expression of $a\cos\theta + b\sin\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$</td></tr>
<tr><td>G2</td><td>Coordinate geometry in two dimensions
• Coordinate geometry of the circle with the equation in the form
$(x - a)^2 + (y - b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$</td></tr>
</table>

Content from O-Level Additional Mathematics	
CALCULUS	
C1	Differentiation and integration • Derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point • Derivative as rate of change • Derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences • Use of Chain Rule • Derivatives of products and quotients of functions • Increasing and decreasing functions • Stationary points (maximum and minimum turning points and points of inflexion) • Use of second derivative test to discriminate between maxima and minima • Connected rates of change • Maxima and minima problems • Integration as the reverse of differentiation • Integration of x^n for any rational n, e^x, $\sin x$, $\cos x$, $\sec^2 x$ and their constant multiples, sums and differences • Integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and e^{ax+b}

SECTION 4: PEDAGOGY

Teaching Processes
Phases of Learning
Teaching Towards Big Ideas
Use of Technology
Blended Learning

4. PEDAGOGY

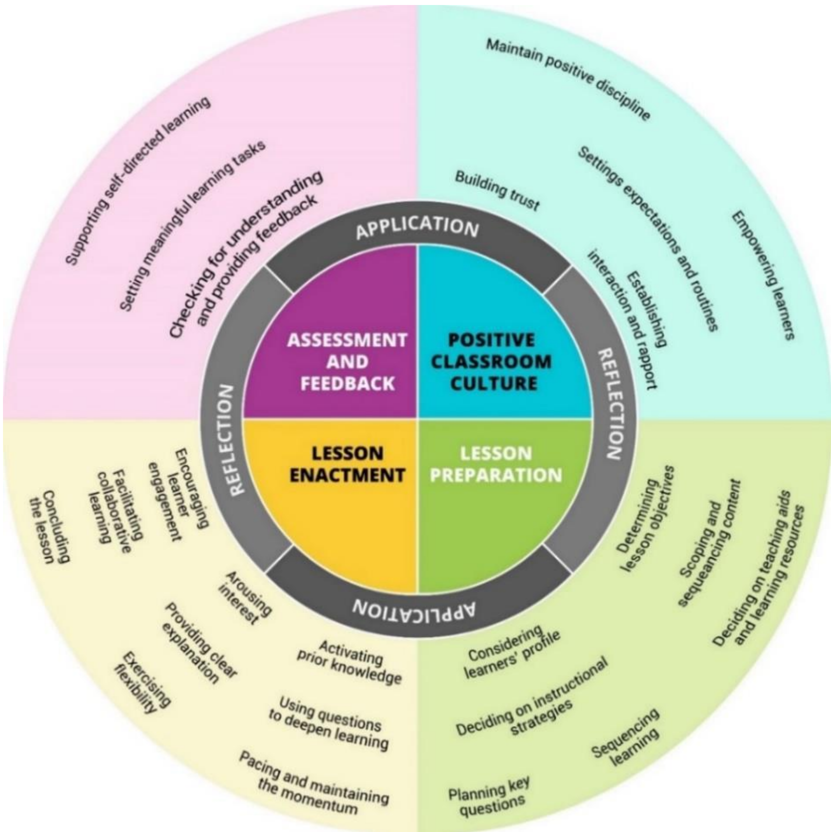
Teaching Processes

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

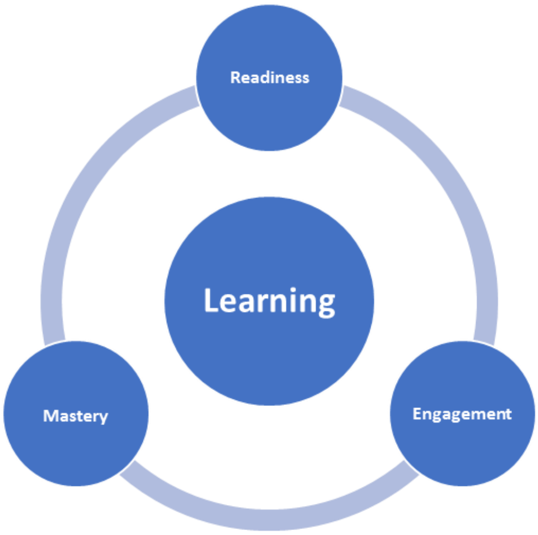
The 4 Teaching Processes are further expanded into 24 Teaching Areas as shown below.



The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.

Phases of Learning

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness*, *engagement* and *mastery*.



Readiness Phase

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (Deciding on Instructional Strategies) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (Exercising Flexibility).

Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (Concluding the Lesson). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

Teaching Towards Big Ideas

To enable students to develop a greater awareness of the nature of mathematics, teachers are encouraged to *teach towards big ideas*, where they help students see and make connections among mathematical ideas within a topic, or between topics across levels or strands. An understanding of big ideas can help students develop a deeper and more robust understanding of mathematics and better appreciation of the discipline.

Teaching towards big ideas requires teachers to be conscious of the big ideas in mathematics that are worth highlighting to their students in each syllabus. For each of these big ideas, they must identify the concepts from different topics, levels and strands that exemplify the big idea. Teachers can develop these concepts as they usually do. However, as they teach these concepts, they should find opportune time to make connections between the concepts (horizontal) and the big idea (vertical). This can be done by explaining the connections, or by guiding students to uncover the connections for themselves by asking questions about related small ideas. Students should develop a lens to look at these big ideas in a way that will facilitate learning of related ideas in future.

Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used by students to carry out investigation (e.g. dynamic geometry software, graphing tools and spreadsheets), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). Getting students who have experience with coding to implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematical concepts as well.

Blended Learning

Blended Learning transforms our students' educational experience by seamlessly blending different modes of learning. The key intents are to nurture: (i) self-directed and independent learners; and (ii) passionate and intrinsically motivated learners.

Blended Learning provides students with a broad range of learning experiences as shown in the diagram below. An aspect of Blended Learning is the integration of *home-based learning (HBL) as a regular feature of the schooling experience*. HBL can be a valuable complement to in-person schooling. Regular HBL can equip students with stronger abilities, dispositions and habits for independent and lifelong learning, in line with MOE's Learn for Life movement.

Examples of Blended Learning Experiences

HBL Days also provide the dedicated time and space for students to actively discover their interests and plan how they should go about pursuing them. Student-initiated learning (SIL) enables students to exercise agency, explore their interests and passions, and learn within and beyond the curriculum.

There are three broad types of SIL activities, namely, school-curated, student-initiated with school facilitation and full student-initiated. Depending on student readiness (e.g. age, disposition, etc.), schools can provide some options for student-initiated learning as scaffolds for those who prefer more guidance at the start, always ensuring that students have agency and choice over what they want to learn. Examples of SIL for A-Level Mathematics are reading a book from the popular maths genre, investigating a problem of interest using open data sources and learning to code.

SECTION 5: ASSESSMENT

Formative and Summative Assessments
National Examinations

5. ASSESSMENT

Formative and Summative Assessments

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. It must be fit-for-purpose.

Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning. The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson. This information should inform the starting point of teaching, the development of the concepts, and the remedial actions that may be necessary.

The purpose of summative assessment or Assessment of Learning (AoL), such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses. It often takes place after learning has been completed, for example, after a topic or a series of topics or at the end of a semester or year. Information from summative assessments can also be used formatively, for instance, to help students close learning gaps and decide on steps which they can take to improve their learning.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning. The feedback must be timely and should focus on both strengths and weaknesses of the work done. Additionally, feedback should include ideas on how students can move forward in their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need

to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.

- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

National Examinations

The first year of examination of H2 Mathematics is 2025.

The assessment objectives (AOs), which reflect the emphases of the syllabus and describe what students should know and be able to do with the concepts and skills learned, is shown below.

Assessment Objectives	Descriptors
AO1	**Use mathematical techniques and procedures** • Recall facts, formulae and notation and use them directly. • Read and use information from tables, graphs, diagrams and texts. • Carry out straightforward mathematical procedures.
AO2	**Formulate and solve problems including those in real-world contexts** • Select relevant mathematical concept or strategy to apply. • Formulate problems into mathematical expressions or models. • Integrate mathematical concepts to solve mathematical problems. • Translate between equivalent forms of mathematical expressions or statements. • Interpret results in the context of a given problem.
AO3	**Reason and communicate mathematically** • Explain the choice of mathematical models or strategies. • Make deductions, inferences and generalisations. • Formulate conjectures and justify mathematical statements. • Construct mathematical arguments and proofs.

Scheme of Examination Papers

Syllabus	Scheme of Examination Papers
H2 Mathematics (9758)	There will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows: **PAPER 1 (3 hours)** A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus. There will be one question on application of Mathematics in real-world contexts, including those from sciences and engineering. This question will

	carry at least 12 marks and may require concepts and skills from more than one topic. Candidates will be expected to answer **all** questions. **PAPER 2 (3 hours)** A paper consisting of two sections, Sections A and B. **Section A** (Pure Mathematics – 40 marks) will consist of 4 to 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus. **Section B** (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus. There will be one question in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. This question will carry at least 12 marks and may require concepts and skills from more than one topic. Candidates will be expected to answer **all** questions.

Further information and details on the national examination are available on the SEAB website.